

Note on Possibility of obtaining a non-relativistic proof of the spin-statistics theorem in the Galilean frame

Gabriel D. Puccini*

*Instituto de Neurociencias, Universidad Miguel Hernández,
Apartado 18, 03550 San Juan de Alicante, Spain.*

Héctor Vucetich

*Observatorio Astronómico, Universidad Nacional de La Plata,
Paseo del Bosque S./N., (1900) La Plata, Argentina*

We replay to the critique by Sudarshan and Shaji of our argument of impossibility to obtain a non-relativistic proof of the spin-statistics theorem in the Galilean frame.

In a recent note [1], Sudarshan and Shaji have presented one objection to the argument of impossibility to obtain a non-relativistic proof of the spin-statistics theorem suggested by us in [2]. To solve the incompatibility between Hermitian field operators and Galilean invariance for massive fields, Sudarshan and Shaji assert that hermiticity can be accomplished by doubling the number of components of $\hat{\xi}_\lambda$ and choosing M as:

$$M = m \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

Finally the authors conclude that “The essential point is that a field (of any spin) can be made to carry an additional charge by doubling the components while still keeping them real. The ‘mass’ M , which may be considered as just another charge, can also be accommodated in an identical fashion. To lament over the Bargmann phase due to M and not worry about any other charge in relation to the proof of the spin-statistics connection stems from assigning M a special status over any other charge that may be relevant to the fields that are being considered”.

Doubling the number of components of the field operators, as is proposed by Sudarshan and Shaji, is the natural way of introducing the concept of charge in the Schwinger formalism for relativistic quantum field theory [5, 6]. To see an example coming from this theory, we suppose that we double the number of components of a given field operator and we choose the charge matrix representation as:

$$Q = e \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

The resulting hermitian field operator will then describe a charged field composed of particles with charges $+e$ and $-e$ (the eigenvalues of Q) [8]. But the price for a such simple introduction of charge is the implicit assumption of crossing symmetry between particles and antiparticles.

It is in this point where relativistic and galilean quantum field theories show their most important differences.

Indeed, mass M is a charge in galilean field theories, but it cannot be treated in the same way as in the relativistic theory because crossing symmetry is not required. In other words, particles ($+m$) and antiparticles ($-m$) cannot be considered on the same footing. This can be easily illustrated for the case of spin-zero field [4]. If we assume the usual commutation or anticommutation rules for the annihilation and creation operators of particles and antiparticles,

$$[\hat{a}(k'), \hat{a}^\dagger(k)]_\mp = \delta(k' - k) , \quad [\hat{b}(k'), \hat{b}^\dagger(k)]_\mp = \delta(k' - k)$$

and we construct a field operator with the correct galilean transformation properties by taking linear combinations of particles annihilation operator and antiparticles creation operator as:

$$\hat{\chi}(\mathbf{x}, t) = (2\pi)^{-3/2} \int d\mu(k) [\alpha e^{\frac{i}{\hbar}(Et - \mathbf{p} \cdot \mathbf{x})} \hat{a}(k) + \beta e^{-\frac{i}{\hbar}(Et - \mathbf{p} \cdot \mathbf{x})} \hat{b}^\dagger(k)] ,$$

we arrive to the following commutation or anticommutation rule:

$$[\hat{\chi}(\mathbf{x}, t), \hat{\chi}^\dagger(\mathbf{y}, t)]_\mp = (|\alpha|^2 \mp |\beta|^2) \delta^3(\mathbf{x} - \mathbf{y}) ,$$

which is satisfied for any value of α and β . In particular equal contribution of particles and antiparticles with $\alpha = \beta$ implies that the commutator vanishes identically. It should be note, moreover, that no spin-statistics relation can be deduced. A complete proof of these two important results of the galilean theory for any spin can be found in [3].

In conclusion, it is not possible to double the number of components of the field operators because it implies to assume equal contribution of particles and antiparticles. And, as we saw, this powerful result of the relativistic theory is not more valid in the galilean theory.

* Electronic address: gpuccini@umh.es

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- [8] The evident symmetry between particles and antiparticles can be shown more clearly by writing Q in diagonal form, but the hermiticity of the fields is lost. This means that in the relativistic theory, the possibility of working with hermitian or non-hermitian field operators can be decided by choosing an appropriated base.